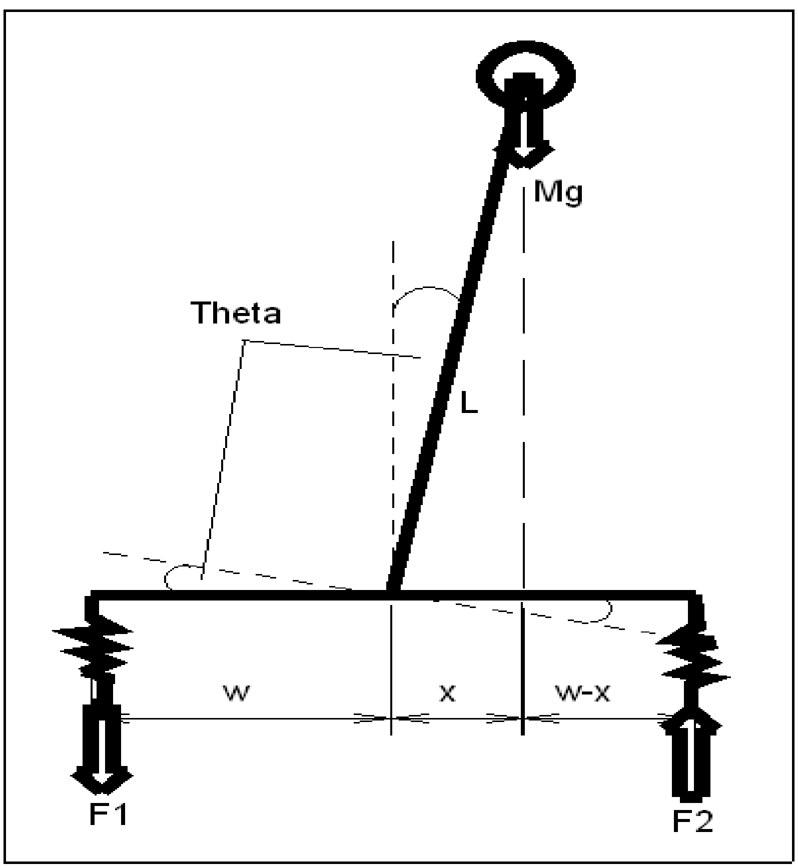
Forensic Engineering Analysis of a Tractor-Trailer Dump Truck that rolled over on the passenger side while in the process of dumping fill (soil) from the body. Based on witness depositions the truck was in forward motion with the body up and approximately 5 cubic yards of fill (soil) was clumped in the front of the body after a major portion of the fill had slid out of the body. The construction access road that the dump truck was on and unloading was moderately compacted by a bulldozer. The dump truck's left rear tire tracks approximately 4 inches deep in the surface of the road. The depression of 4 inches is not uncommon on construction sites. Plaintiff claims that the left rear tire depression which plaintiff claims was the roadway sinking under the load of the truck caused the dump truck to overturn. Plaintiff did not give an estimate of forward dump truck speed after dumping and before overturning.

Analysis: The forward motion of the dump truck while the body was at the maximum dumping position, (a higher center of gravity of the truck), created an inverted pendulum effect on the raised body. The speed that the dump truck pulls forward directly affected the sideways (angular) motion of the raised body.

The Dynamic Torque created by the inverted pendulum effect would equal or exceed the Static Torque (Moment) of the raised body of the vehicle. The Dynamic Torque (Moment) is a function of the angular acceleration created by the sideways motion of the raised truck body. This created a rotation of the dump truck body to the passenger side.

The dynamic inverted pendulum motion of the raised body created an overturning motion on the raised body.

Below is the derivation of the Torque created by the Angular Moment of Inertia of the raised dump truck body.



F_1 and F_2 are equal and opposite modeled spring forces resisting the overturning moment.
T = Torque created by the off-center of gravity of the truck.
w = half the width of the truck body.
x = some distance from the mid-point of the truck.
θ = is some angle that the truck tips.
θ' = angular velocity, omega (ω).
θ'' = angular acceleration, alpha (α)
α = angular acceleration of the truck body angular motion.
b = damping coefficient of the truck rear springs.
k = spring constant of the truck rear springs.
I = Moment of Inertia of the truck about the tire/ground contact.
L = Length of the cg above the ground.

Solution: Counterclockwise; Down; and Left are positive.

$$T = F\cdot w + F\cdot w - M\cdot g\cdot x$$

$$2F = -2b\cdot y' - 2k\cdot y = -2b\cdot\theta'\cdot L - 2k\cdot L\cdot\sin(\theta)$$

$$T = 2\cdot F\cdot w - M\cdot g\cdot x$$

$$T = -2\cdot b\cdot\theta'\cdot w\cdot L - 2k\cdot w\cdot L\cdot\sin(\theta) - M\cdot g\cdot L\cdot\sin(\theta) = I\cdot\alpha$$

For small angles $\sin(\theta) = \theta$

$$I\cdot\theta''(t) + 2\cdot b\cdot w\cdot L\cdot\theta'(t) + 2\cdot k\cdot w\cdot L\cdot\theta(t) + M\cdot g\cdot L\cdot\theta(t) = 0$$

$$\alpha = \theta''$$

$$I\cdot\theta''(t) + 2\cdot b\cdot w\cdot L\cdot\theta'(t) + (2\cdot k\cdot w\cdot L + M\cdot g\cdot L)\cdot\theta(t) = 0$$

$$\theta''(t) + \frac{2\cdot b\cdot w\cdot L}{I}\cdot\theta'(t) + \frac{(2\cdot k\cdot w\cdot L + M\cdot g\cdot L)}{I}\cdot\theta(t) = 0$$

$$2\cdot\gamma = \frac{2\cdot b\cdot w\cdot L}{I}$$

$$\theta''(t) + 2\cdot\gamma\cdot\theta'(t) + \omega_o^{\,2}\cdot\theta(t) = 0$$

$$\omega_o^{\,2} = \frac{(2\cdot k\cdot w\cdot L + M\cdot g\cdot L)}{I}$$

$$R^2\cdot e^{R\cdot t} + 2\cdot\gamma\cdot e^{R\cdot t} + \omega_o^{\,2}\cdot e^{R\cdot t} = 0$$

Try: $\theta = e^{R\cdot t}$

$$R = -\gamma + \sqrt{\gamma^2 - \omega_o^{\,2}} \qquad R = -\gamma - \sqrt{\gamma^2 - \omega_o^{\,2}}$$

$$\theta' = R\cdot e^{R\cdot t}$$

$$\theta'' = R^2\cdot e^{R\cdot t}$$

$$\theta(t) = A\cdot e^{-\gamma+\sqrt{\gamma^2-\omega_o^{\,2}}\cdot t} + B\cdot e^{-\gamma-\sqrt{\gamma^2-\omega_o^{\,2}}\cdot t}$$

Let: $\kappa^2 = \gamma^2 - \omega_o^{\,2}$

$$-\kappa^2 = \omega_o^{\,2} - \gamma^2$$

$$\theta(t) = A\cdot e^{(-\gamma+i\cdot\kappa)\cdot t} + B\cdot e^{(-\gamma-i\cdot\kappa)\cdot t}$$

$$i\cdot\kappa = \sqrt{\omega_o^{\,2} - \gamma^2}$$

$$\theta(t) = A\cdot e^{-\gamma\cdot t}\cdot\cos(\kappa\cdot t) + B\cdot e^{-\gamma\cdot t}\cdot\sin(\kappa\cdot t)$$

$$\theta(t) = e^{-\gamma\cdot t}\cdot(A\cdot\sin(\kappa\cdot t) + B\cdot\cos(\kappa\cdot t))$$

Apply initial conditions where at t = 0; θ = 4"/48" = 1/12 rad and θ' = 0, then:

$$B = \frac{1}{12}$$

$$\theta(t) = e^{-\gamma \cdot t} \cdot \left(A \cdot \sin(\kappa \cdot t) + \frac{1}{12} \cdot \cos(\kappa \cdot t) \right)$$

$$\theta'(t) = e^{-\gamma \cdot t} \cdot \left(A \cdot \kappa \cdot \cos(\kappa \cdot t) - \frac{1}{12} \cdot \kappa \cdot \sin(\kappa \cdot t) \right) - \gamma \cdot e^{-\gamma \cdot t} \cdot \left(A \cdot \sin(\kappa \cdot t) + \frac{1}{12} \cdot \cos(\kappa \cdot t) \right)$$

$$0 = A \cdot \kappa - \frac{\gamma}{12} \qquad A = \frac{\gamma}{12\kappa}$$

$$\theta(t) = \frac{e^{-\gamma \cdot t}}{12} \cdot \left(\frac{\gamma}{\kappa} \cdot \sin(\kappa \cdot t) + \cos(\kappa \cdot t) \right)$$

Where ω_o is not = to ω. ω is the angular velocity of the raised trailer body and ω_o is the natural frequency of the system.

Approximate dimensions of tractor and frame:

M_t = Mass of the tractor.
h_t = height of cg of tractor.
l_t = length of tractor.
M_f = Mass of frame.
h_f = height of cg of frame.
l_f = lenght of frame.

M_b = Mass of the body.
h_b = height of cg of body.
l_b = length of body.
M_s = Mass of soil in stuck in body.
h_s = height of cg of soil in body.
l_s = length of soil in body.

For the tractor trailer to turn over, the center of mass must move over the edge of the tractor trailer or the center of mass must move 4 feet or half the width of the tractor trailer. Therefore theta (θ) or the angle of oscillation must be half the width divided by vertical height of the cg.

Determine the geometry, Moments of Inertia, weight of the Tractor-Trailer dump truck.
Body is 36 feet long with capacity 54 cy; the body is approximately 5 feet above the ground.

$$M_t := \frac{15000\,lb}{g} \qquad l_t := 30ft \qquad h_t := 3ft$$

$$M_f := \frac{7000}{g} lb \qquad l_f := 35ft \qquad h_f := 4ft$$

$$M_b := \frac{7000\,lb}{g} \qquad l_b := 35ft \qquad h_b := 5ft$$

$$d_s := 100 \frac{lb}{ft^3} \qquad l_s := 6ft \qquad h_s := 4ft \qquad w_s := 6ft$$

$$M_s := \frac{d_s \cdot h_s \cdot l_s \cdot w_s}{g} \qquad M_s = 448 \frac{lb \cdot s^2}{ft} \qquad W_s := d_s \cdot l_s \cdot h_s \cdot w_s \qquad W_s = 14400\,lb$$

The estimated Moments of Inertia of the elements relative to the ground;

$$I_t := \frac{1}{12} \cdot M_t \cdot l_t^{\,2} + M_t \cdot \left(h_t\right)^2 \qquad I_t = 39162\,\mathrm{lb \cdot ft \cdot s^2}$$

$$I_f := \frac{1}{12} \cdot M_f \cdot l_f^{\,2} + M_f \cdot h_f^{\,2} \qquad I_f = 25691\,\mathrm{lb \cdot ft \cdot s^2}$$

$$I_b := \frac{1}{12} \cdot M_b \cdot l_b^{\,2} + M_b \cdot \left(h_b + h_f\right)^2 \qquad I_b = 39833\,\mathrm{lb \cdot ft \cdot s^2}$$

$$I_s := \frac{1}{12} \cdot M_s \cdot l_s^{\,2} + M_s \cdot \left(h_s + h_f\right)^2 \qquad I_s = 29987\,\mathrm{lb \cdot ft \cdot s^2}$$

$$I := I_t + I_f + I_b + I_s \qquad I = 134673\,\mathrm{lb \cdot ft \cdot s^2}$$

The center of mass of the raised body relative to the ground (h), say @ 45 degrees:

$$L := \left[\frac{M_t \cdot \frac{h_t}{2} + M_f \cdot h_f + M_b \cdot \left(\frac{h_b}{2} + l_f \cdot \sin(45\mathrm{deg})\right) + M_s \cdot \left(\frac{h_s}{2} + l_f \cdot \sin(45\mathrm{deg})\right)}{M_t + M_f + M_b + M_s}\right]$$

Height of center of mass of the system.

$$L = 14.4\,\mathrm{ft}$$

The Dynamic Torques due to the angular acceleration must exceed the Static Torque to overturn the truck. Since the general Torque equation is a function of time and directly proportional to time (T), time (T) is increased until the Dynamic Torque exceeds the Static Torque.

t = a range of time from zero to (T) which is necessary to generate the curves.
T = is the exact time it takes for the dynamic Torque to exceed the Static Torque and overturn the truck.

$$M := \left(M_t + M_f + M_b + M_s\right) \qquad M = 1349\,\frac{\mathrm{lb \cdot s^2}}{\mathrm{ft}} \qquad W := M \cdot g \qquad W = 43400\,\mathrm{lb} \qquad w := 4\mathrm{ft}$$

$$k := \frac{85000}{g}\,\frac{\mathrm{N}}{\mathrm{m}} \qquad k = 5824\,\frac{\mathrm{lb}}{\mathrm{ft}}$$

$$b := \frac{9592}{g}\,\frac{\mathrm{N \cdot s}}{\mathrm{m}} \qquad b = 657\,\frac{\mathrm{lb \cdot s}}{\mathrm{ft}}$$

This Omega(ω_o) is the frequency of the system not the angular velocity.

$\omega_o := \sqrt{\dfrac{(2 \cdot k \cdot w \cdot L + M \cdot g \cdot L)}{I}}$ $\omega_o = 3.11 \dfrac{1}{s}$ $T := \dfrac{2\pi}{\omega_o}$ $T = 2.02\,s$ $f := \dfrac{1}{T}$ $f = 0.49 \cdot Hz$

$\gamma := \dfrac{b \cdot w \cdot L}{I}$ $\gamma = 0.282 \dfrac{1}{s}$ $\kappa := \sqrt{\omega_o^{\,2} - \gamma^2}$ $\kappa = 3.09 \dfrac{1}{s}$

$t := 0s, 0.01s .. \dfrac{T}{2}$

$\theta(t) := \dfrac{e^{-\gamma \cdot t}}{12} \cdot \left(\dfrac{\gamma}{\kappa} \cdot \sin(\kappa \cdot t) + \cos(\kappa \cdot t)\right)$

$\theta(t) =$

4.8
4.8
...

$\cdot deg$

$\theta'(t) := \dfrac{d}{dt}\theta(t)$

$\theta'(t) =$

$6.12 \cdot 10^{-15}$
-0.01
-0.02
...

$\dfrac{1}{s}$

$\theta''(t) := \dfrac{d^2}{dt^2}\theta(t)$

$\theta''(t) =$

-0.8
-0.8
-0.79
...

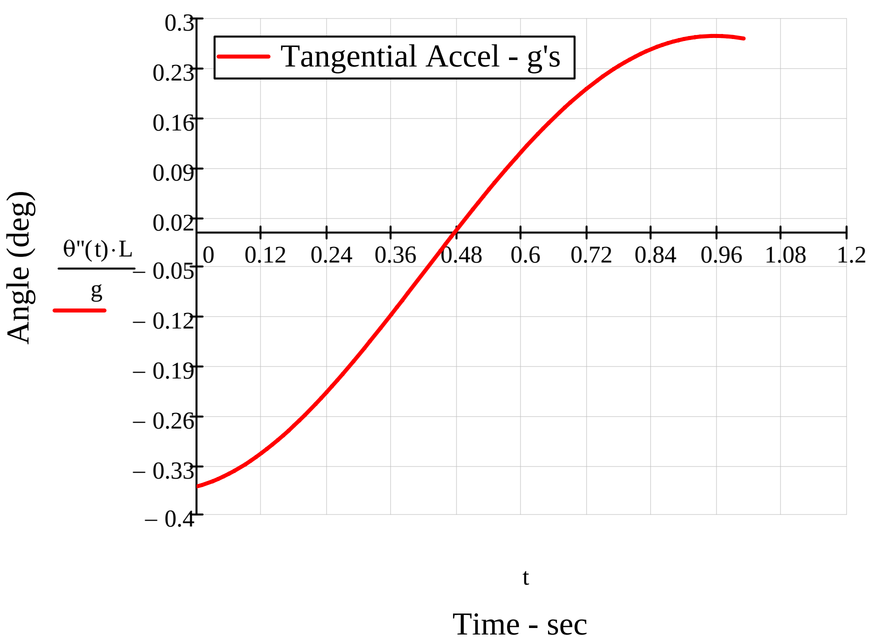
$\dfrac{1}{s^2}$

Find the tangential acceleration when the body swings back from one side to the other, or 1/2 of the period (T).

$q := \dfrac{T}{2}$

$a_T := \theta''(q) \cdot L$ $a_T = 0.27 \cdot g$ Tangential acceleration.

$\mathrm{asin}(\theta(q)) = -3.6 \cdot deg$ $\theta(q) = -3.6 \cdot deg$ Angle of trailer motion.



The total force to cause overturning when the weight at the center of gravity swings back in half the period pendulum motion to the other side, creates a tangential acceleration at the cg, plus the weight of the off-center center of gravity.

If the dynamic torque is greater than the static torque then the trailer will overturn.

$$W \cdot L \cdot \frac{a_T}{g} > W \cdot w + W \cdot L \cdot \sin(\theta(q))$$

$$L \cdot \left(\frac{a_T}{g} - \sin(\theta(q)) \right) > w$$

$$L \cdot \left(\frac{a_T}{g} - \sin(\theta(q)) \right) = 4.82\,\text{ft}$$ Which is greater than $w = 4\,\text{ft}$

The dynamic force created an equivalent lever arm which is greater than half the width of the trailer, resulting in the center of gravity moving over the edge of the trailer and therefore the trailer will turn over.